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Network-based precise tracking control of systems subject to stochastic failure and non-zero input

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Abstract: This study deals with the problem of reliable precise tracking control for the networked control system with non-zero external inputs (NEIs). A new control scheme is developed by introducing an integral item in the control law and taking stochastic actuator failure (SAF) into account, by which a better steady tracking performances can be achieved. Based on Lyapunov theory, a reliable control design method that guarantees the network-based tracking control systems with zero steady-state error in the conditions of both NEIs and SAFs is designed. Simulation results demonstrate the effectiveness of the developed controller design scheme.

1 Introduction

With the rapid development of network technologies, more and more control systems are implemented over communication networks, which is called networked control system (NCS). In the past few decades, much attention has been drawn to the researches on the stability and stabilisation of the NCS for its predominant advantages (such as low cost, reduced weight and power requirements, simple installation and maintenance and high reliability). Considerable attention and effort have been paid to its challenging issue (such as network-induced delay, packet losses); see [1–5] and reference their in.

Since tracking control has wide applications in dynamic process, such as remote robot control, flight control, processing control, etc., however, it is hard to achieve a good tracking control performance by using classic controller when the control signal is transmitted via a network. Therefore the network-based tracking control system (NTCS) has drawn a great deal of interest to the researchers [6–10] so far. The objective of the tracking control for the designers is to drive the output of the plant to follow the predefined trajectory precisely when the system is under external inputs. In comparison with the problem of stabilisation control, the tracking control is therefore relatively difficult to handle [9, 11], especially for the NTCS. The existed approach on tracking control is mainly focused on adaptive control methods [7, 12] and H_∞ control technologies [6, 9]. However, in practise, it may waste a large amount of CPU resources, and it even leads to non-real-time control; while using H_∞ control technology, generally, requires the external inputs belonging to the set of $l_2[0\ \infty)$. Although it can achieve a certain H_∞ disturbance attenuation level, it is hard to meet the practical requirement, since either the input of the reference model or the external disturbance does not belong to the set $l_2[0\ \infty)$ in most cases. Moreover, it should be pointed out that there exists a big steady-state error (SSE) when the inputs of the system are of non-zero external inputs (NEIs) by using H_∞ design method, that is, the capability of disturbance rejection for this method is weak. To the best of our knowledge, unfortunately, few works pay attention to the steady tracking performances (STP) of the NTCS, although the problem of STP should be concerned primarily for the tracking control systems. This motivates us the present study.

In most practical control systems, components' failure (including sensors, actuators and even the plant itself) may occur at an uncertain time. The fault may lead to the performance of the system deterioration or even the instability. Fault tolerance means the ability of the system maintaining its stability and performance in spite of unknown faults within the system. Therefore fault-tolerant control can guarantee the system stability not only during normal operations but also under an abnormal situation, which is especially important for precise tracking control systems. The existed results on the fault components mainly focus on a certain case [13] (completely failure or partly failure) or a class of uncertain case [14–16] (fault varying within a known interval). Very few works study the case of components of the system that are subject to a certain stochastic failure.

In this paper, we aim to develop a fault-tolerant controller for NTCS with both stochastic actuator failures (SAFs) and

NEIs. The main contributions of the paper are as follows. First, a new control scheme is developed by introducing an integral item to remove the offsets between the output of the plant and the output of the reference model when the system is of non-zero input, that is, the system have an excellent STP by using the proposed control law when the system is of non-zero input. Second, a stochastic actuator fault model described by a rand matrix is established, which reflects the real actuator fault closely and covers several classes of well-studied models. Finally, a reliable control design method for such NTCS is developed in terms of linear matrix inequalities (LMIs), such that the NTCS has a good STP in conditions of both NEIs and SAFs.

The remainder of the paper is organised as follows. The problem formulation is given in Section 2. The reliable control design method is provided in Section 3. Section 4 presents the design results and simulations. Finally, the study's findings are summarised in Section 5.

Notation: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $\mathbb{R}^{n\times m}$ is the set of real $n \times m$ matrices; I is the identity matrix of appropriate dimensions; $\|\cdot\|$ stands for the Euclidean vector norm or spectral norm as appropriate. The notation $X > 0$ (respectively, $X < 0$), for $X \in \mathbb{R}^{n\times n}$ means that the matrix X is a real symmetric positive definite (respectively, negative definite); when x is a stochastic variable, $\mathbb{E}\{x\}$ stands for the expectation of x; the asterisk $*$ in a matrix is used to denote term that is induced by symmetry.

2 Problem formulation

As shown in Fig. 1, our aim in this paper is to design a controller such that the output of the plant tracks the trajectory of output of the reference model when the system with NEIs and SAFs.

Suppose the physical plant model is given by a linear discrete-time system

$$x(k+1) = Ax(k) + Bu^{\mathrm{F}}(k) + B_{\omega}\omega(k) \tag{1}$$

$$y(k) = Cx(k) \tag{2}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $u^{\mathrm{F}}(k) \in \mathbb{R}^m$ is the control input subject to SAFs, $y(k) \in \mathbb{R}^q$ is the output vector.

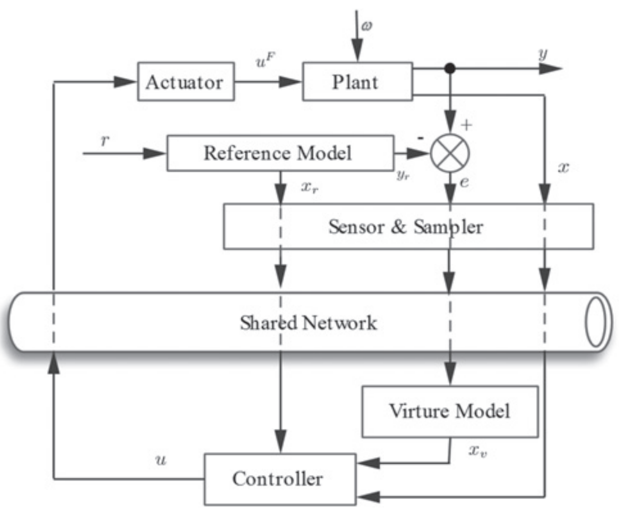

Fig. 1 *Schematic diagram of NTCS*

A, B, B_{ω} and C are system matrices with appropriate dimensions, and $\omega(k)$ is a non-zero disturbance, which takes the form of

$$\lim_{k\to\infty} \omega(k) = c \tag{3}$$

where c is an unknown non-zero constant.

The tracked plant is represented by the following reference model

$$x_{\mathrm{r}}(k+1) = A_{\mathrm{r}}x_{\mathrm{r}}(k) + B_{\mathrm{r}}r(k) \tag{4}$$

$$y_{\mathrm{r}}(k) = C_{\mathrm{r}}x_{\mathrm{r}}(k) \tag{5}$$

where $x_{\mathrm{r}}(k) \in \mathbb{R}^{n_{\mathrm{r}}}$ is the reference state, $y_{\mathrm{r}}(k) \in \mathbb{R}^{n_q}$ is the output vector with a same dimension of $y(k)$, $r(k) \in \mathbb{R}^{n_{\mathrm{r}}}$ is the reference input vector. A_{r} is a specified asymptotically stable matrix, B_{r} and C_{r} are known matrices with appropriate dimensions.

To facilitate theoretical development, we make the following assumptions for the NTCS.

Assumption 1: Sensors are clock-driven, while actuators and controller are event-driven, moreover the clocks among all the devices are synchronised.

Assumption 2: As shown in Fig. 1, the signals of $x(k), x_{\mathrm{r}}(k)$ and $e(k)$ transmitted with a signal packet are online measurable.

In order to track the specific signal with an excellent steady-state performance, the following controller law is proposed

$$u(k) = K_1x(i_k) + K_2x_{\mathrm{r}}(i_k) + K_3\sum_{s=0}^{i_k-1} e(s) \quad \lceil \tau_k + i_k \rceil \le k \le \lceil \tau_{k+1} + i_{k+1} \rceil \tag{6}$$

where $K_i (i = 1, 2, 3)$ are the controller gains to be determined. $e(k) = y(k) - y_{\mathrm{r}}(k)$. τ_k is the network-induced delay, and i_k is the kth sampling instant at sensor side. The set $\{i_1, i_2, \ldots, \}$ is a subset of the set $\{1, 2, \ldots\}$. If $i_{k+1} \ge i_k + 1$, it means that some packets are lost. For example, as is shown in Fig. 2, the packet at $k = 6$ is lost, and wrong sequence occurs between $k = 3$ and 4, then the packet at $k = 3$ is discarded. The controller uses the effective packets in Fig. 2 at $k = 2, 4, 5, 7$ are in the intervals $\mathcal{I}_1, \mathcal{I}_2, \mathcal{I}_3$ and $\mathcal{I}_4$, respectively. Obviously $\cup_{i=0}^{\infty}\mathcal{I}_i = [k_0, \infty)$, where k_0 is an integer.

Define $d_k = k - i_k$, it leads to

$$0 < \tau_k \le d_k \le \lceil \tau_{k+1} + (i_{k+1} - i_k) \rceil \le d_M \tag{7}$$

Then (6) can be written as

$$u(k) = K_1x(k - d_k) + K_2x_{\mathrm{r}}(k - d_k) + K_3 \sum_{s=0}^{k-d_k-1} e(s) \tag{8}$$

Remark 1: Supposing the physical system (1) is stable, one can see that the actuator will keep working till the error between $y(k)$ and $y_{\mathrm{r}}(k)$ reduces to zero, because if the error is non-zero, the control force will be further strengthen over time because of the integrated item in (6). Therefore, a better STP can be obtained by using this control scheme in comparison with using a simple static state feedback controller or static output feedback controller.

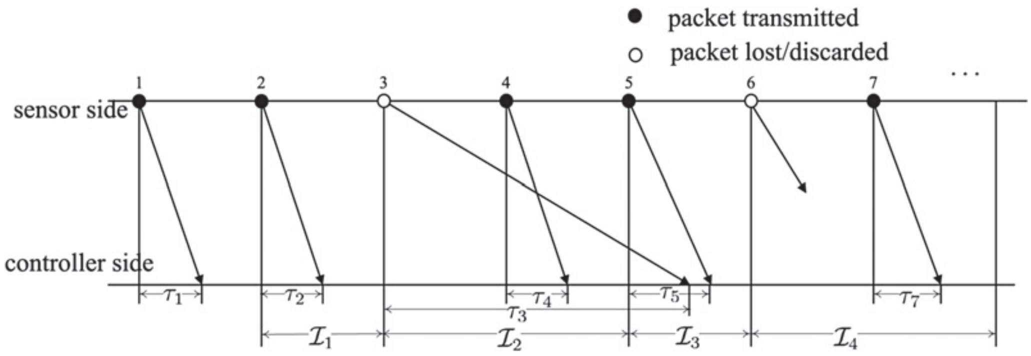

Fig. 2 *Timing diagram for data transmission*

For technical convenience, here we construct a new state variable $x_v(k) \in \mathbb{R}^{n_v}$ as follows

$$x_v(k+1) = x_v(k) + e(k) \tag{9}$$

$$x_v(k) = 0, \quad -d_M \le k \le 0 \tag{10}$$

Recalling the definition of $e(k)$, we have

$$x_v(k+1) = x_v(k) + Cx(k-d_k) - C_r x_r(k-d_k) \tag{11}$$

$$\sum_{i=0}^{k-d_k-1} e(s) = x_v(k-d_k) \tag{12}$$

Then (8) can be further rewritten as

$$u(k) = K_1 x(k-d_k) + K_2 x_r(k-d_k) + K_3 x_v(k-d_k) \tag{13}$$

Here we proposed a stochastic actuator fault model as

$$u^F(k) = \Theta u(k) \tag{14}$$

where $\Theta = \text{diag}\{\theta_1, \theta_2, \ldots, \theta_m\}$ is a rand matrix, and θ_i with its expectation $\bar{\theta}_i$ and variance σ_i $(i \in \{1, 2, \ldots, m\} \triangleq \mathcal{F})$, respectively, denotes the stochastic failure in each channels of the actuators.

Remark 2: The physically means of the actuator fault can be reflected more closely by (14) whose idea is borrowed from the model of missing measurement in [17]. Moreover, the stochastic actuator fault model in (14) covers several class of well-studied actuator failure model. For example, if one let $\theta_i \equiv 0$ or $\theta_i \equiv 1 (i \in \mathcal{F})$, it means the actuator in each channels is complete failure or intactness, respectively [13, 18]. While $\theta_i \in (0\ 1)(i \in \mathcal{F})$, it denotes the actuator is of partial failure [16, 19].

Combining (1), (4), (9), (13) and (14), we can obtain the following augmented closed-loop system

$$\xi(k+1) = \bar{A}\xi(k) + \tilde{A}_d \xi(k-d_k) + \bar{B}v(k) \tag{15}$$

where $\bar{A} = \begin{bmatrix} A & 0 & 0 \\ 0 & A_r & 0 \\ 0 & 0 & I \end{bmatrix}$, $\tilde{A}_d = \begin{bmatrix} B\Theta K_1 & B\Theta K_2 & B\Theta K_3 \\ 0 & 0 & 0 \\ C & -C_r & 0 \end{bmatrix}$, $\xi(k) = \begin{bmatrix} x(k) \\ x_r(k) \\ x_v(k) \end{bmatrix}$, $v(k) = \begin{bmatrix} \omega(k) \\ r(k) \\ 0 \end{bmatrix}$, $\bar{B} = \text{diag}\{B_\omega, B_r, 0\}$.

3 Fault-tolerant control design

In this section, our main interest is to develop a fault-tolerant controller such that the output of the plant y can track the specific trajectory y_r with an excellent STP under the conditions of both NEIs and SAFs.

3.1 Analysis of stability for NTCS

As is stated in Remark 1, if the system is stable, it gives rise to a good tracking performance thanks to the proposed controller in (14). Hence, in this subsection, we will develop a criteria of asymptotic stability for the closed-loop NTCS with $v(k) \equiv 0$ firstly.

For the sake of technical simplicity, we rewrite (15) with $v(k) = 0$ as

$$\xi(k+1) = \mathcal{A}\eta(k) + \tilde{\mathcal{A}}\eta(k) \tag{16}$$

$$\eta(k) = \varphi(k) - d_M \le k \le 0 \tag{17}$$

where $\mathcal{A} = [\bar{A}\ \bar{A}_d\ 0]$, $\tilde{\mathcal{A}} = [0\ \hat{A}_d\ 0]$, $\eta(k) = [\xi^T(k)\ \xi^T(k-d_k)\ \xi^T(k-d_M)]^T$, and

$$\bar{A}_d = \begin{bmatrix} B\bar{\Theta}K_1 & B\bar{\Theta}K_2 & B\bar{\Theta}K_3 \\ 0 & 0 & 0 \\ C & -C_r & 0 \end{bmatrix}, \quad \bar{\Theta} = \mathbb{E}\{\Theta\}$$

$$\hat{A}_d = \begin{bmatrix} B(\Theta - \bar{\Theta})K_1 & B(\Theta - \bar{\Theta})K_2 & B(\Theta - \bar{\Theta})K_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Before starting our main results, we first introduce the following definition and lemma.

Definition 1: System (16) is said to be mean-square asymptotically stable (MSAS), if there exists a scalar $\varepsilon > 0$, such that

$$\mathbb{E}\left\{\sum_{k=0}^{\infty} \|\eta(k)\|^2\right\} \le \varepsilon \mathbb{E}\{\|\varphi(k)\|^2\} \tag{18}$$

Lemma 1 [20]*:* For symmetric positive-definite matrix $R \in \mathbb{R}^{n \times n}$, a function $0 \le d_k \le d_M$ and a vector function $\xi(k) \in \mathbb{R}^n$, $\varsigma(k) = \xi(k+1) - \xi(k)$, such that the following integration is well defined, it holds that

$$-d_M \sum_{i=k-d_M}^{k-1} \varsigma^T(i) R \varsigma(i) \le \eta^T(k) \begin{bmatrix} -R & * & * \\ R & -2R & * \\ 0 & R & -R \end{bmatrix} \eta(k) \tag{19}$$

where $\eta(k) = [\xi^T(k)\ \xi^T(k-d_k)\ \xi^T(k-d_M)]^T$.

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Theorem 1: For given matrix $\bar{\Theta}$ and scalars $\sigma_i (i \in \digamma)$, system (16) is said to be MSAS if there exist matrices $P > 0, Q > 0, R > 0$ and $K_j (j = 1, 2, 3)$ with appropriate dimensions, such that

$$\Xi = \begin{bmatrix} \Xi_{11} & * & * \\ \Xi_{21} & -\Xi_{22} & * \\ \bar{\mathcal{A}}_F & 0 & -\Xi_{33} \end{bmatrix} < 0 \tag{20}$$

holds, where

$$\Xi_{11} = \begin{bmatrix} -P + Q - R & * & * \\ R & -2R & * \\ 0 & R & -Q - R \end{bmatrix}$$

$$\Xi_{21} = \begin{bmatrix} \mathcal{A}^T & d_M \hat{\mathcal{A}}^T \end{bmatrix}^{\mathrm{T}}, \Xi_{22} = \mathrm{diag}\{P^{-1}, R^{-1}\}$$

$$\Xi_{33} = \mathrm{diag}\{\underbrace{P^{-1}, \ldots, P^{-1}}_{m}, \underbrace{R^{-1}, \ldots, R^{-1}}_{m}\}$$

$$\hat{\mathcal{A}} = \begin{bmatrix} (\bar{\mathcal{A}} - I) & \bar{\mathcal{A}}_d & 0 \end{bmatrix},$$

$$\bar{\mathcal{A}}_F = \begin{bmatrix} \frac{1}{d_M} \bar{\mathcal{A}}^T_{d_{F_1}} & \cdots & \frac{1}{d_M} \bar{\mathcal{A}}^T_{d_{F_m}}, & \bar{\mathcal{A}}^T_{d_{F_1}} & \cdots & \bar{\mathcal{A}}^T_{d_{F_m}} \end{bmatrix}^{\mathrm{T}}$$

$$\bar{\mathcal{A}}_{d_{F_i}} = \begin{bmatrix} 0 & \sigma_i d_M A_{d_{Fi}} & 0 \end{bmatrix} \quad (i \in \digamma)$$

$$A_{d_{Fi}} = \begin{bmatrix} BF_iK_1 & BF_iK_2 & BF_iK_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} (i \in \digamma)$$

Proof: Defining $F_i = \mathrm{diag}\{\underbrace{0, \ldots, 0}_{i-1}, 1, \underbrace{0, \ldots, 0}_{m-i}\}$, and taking the definition of Θ in (14) into consideration, one can easily know that

$$\mathbb{E}\{\tilde{\mathcal{A}}\} = 0 \tag{21}$$

$$\mathbb{E}\{\hat{A}_d^T \Pi \hat{A}_d\} = \sum_{i=1}^{m} \sigma_i^2 A^T_{d_{Fi}} \Pi A_{d_{Fi}} \tag{22}$$

where Π is a symmetrical positive-definite matrix.

Let us define a Lyapunov function for system (16) as

$$V(k) = \xi^T(k) P \xi(k) + \sum_{i=k-d_M}^{k-1} \xi^T(i) Q \xi(i) + d_M \sum_{i=-d_M}^{-1} \sum_{j=k+i}^{k-1} \varsigma^T(j) R \varsigma(j) \tag{23}$$

where $\varsigma(k) = \xi(k+1) - \xi(k)$, Combining (21) and (22) and Lemma 1, we have (see equation at the bottom of the page)

Recalling (20) and using Schur complement, there exists a constant $\iota > 0$, such that

$$\mathbb{E}\{\Delta V(k)\} \le -\iota \mathbb{E}\{\eta^T(k)\eta(k)\} \le -\iota \mathbb{E}\{\xi^T(k)\xi(k)\} \tag{24}$$

Since $\cup_{k=1}^{\infty} [\tau_k + i_k, \tau_{k+1} + i_{k+1}] = (0, \infty)$, we have

$$\mathbb{E}\left\{\sum_{k=0}^{\infty} \xi^T(k)\xi(k)\right\} \le \iota^{-1} \mathbb{E}\{V(0)\} \tag{25}$$

From the construction of $V(k)$, we can conclude that there exists a constant $\varepsilon > 0$, such that

$$\mathbb{E}\{V(0)\} \le \iota\varepsilon \mathbb{E}\{\varphi^T(k)\varphi(k)\} \tag{26}$$

Based on the definition of the MSAS, the proof is completed. □

3.2 Analysis of STP with consideration of NEIs

According to (9), we have

$$\lim_{k\to\infty} \mathbb{E}\{x_v(k+1)\} = \lim_{k\to\infty} \mathbb{E}\{(x_v(k) + e(k))\} \tag{27}$$

Theorem 1 gives the condition of MSAS for the augmented system (16), which means the subsystem (9) is also MSAS if (20) holds. Under this condition, we have

$$\lim_{k\to\infty} \mathbb{E}\{x_v(k+1)\} = \lim_{k\to\infty} \mathbb{E}\{x_v(k)\} \tag{28}$$

It is obviously that

$$\lim_{k\to\infty} \mathbb{E}\{e(k)\} = 0 \tag{29}$$

Recalling the definition of $e(k)$, one can easily know from (29) that the steady requirement of tracking control can be achieved by using the controller (14) even though the disturbance is a NEI.

Remark 3: In fact, the controller (8) is a combination of state- and output-feedback control, moreover, it borrows the ideas of the proportional-integral (PI) control in the classic control theory, that is, the last item of (8) can let the systems produce zero SSE.

3.3 Controller design for NTCS

The stability condition of NTCS is given in Theorem 1, however, the controller parameters (14) cannot be obtained directly by using LMI toolbox because of the matrices P, P^{-1}, R and R^{-1} existed in the same inequality (20). The following iterative algorithm [21] is used to find the feasible solution.

Algorithm:
Step 1: Introduce two new variables U and V, then replace P^{-1} and R^{-1} with those two matrices in (20), respectively.

$$\mathbb{E}\{\Delta V(k)\} \le \mathbb{E}\Bigg\{\xi^T(k)(-P+Q)\xi(k) - \xi^T(k-d_M)Q\xi(k-d_M) + \eta^T(k)\mathcal{A}^T P \mathcal{A}\eta(k) + d_M^2 \eta^T(k)\hat{\mathcal{A}}^T R \hat{\mathcal{A}}\eta(k) + \eta^T(k)\begin{bmatrix} -R & * & * \\ R & -2R & * \\ 0 & R & -R \end{bmatrix}\eta(k) + \sum_{i=1}^{m} \xi^T(k-d_k) A^T_{d_{Fi}} (\sigma_i^2 P + d_M^2 \sigma_i^2 R) A_{d_{Fi}} \xi(k-d_k)\Bigg\}$$

A new matrix is defined by $\bar{\Xi}$. The condition (20) then turns to

$$\begin{cases} \bar{\Xi} < 0 \\ PU = RV = I \end{cases} \tag{30}$$

Step 2: Find a feasible solution P_0, R_0, S_0, T_0 to LMI (31), set $\kappa = 0$.

$$\begin{cases} \bar{\Xi} < 0 \\ \begin{bmatrix} P & * \\ I & U \end{bmatrix} \geq 0, \ \begin{bmatrix} R & * \\ I & V \end{bmatrix} \geq 0 \end{cases} \tag{31}$$

Step 3: Solve the following LMI problem for the variables P, R, U, V

$$\begin{cases} \min \ \text{Trace}(P_\kappa U + R_\kappa V + U_\kappa P + V_\kappa R) \\ \text{subject to LMI in (31)} \end{cases} \tag{32}$$

Step 4: Select a small enough $\varrho(\varrho > 0)$, if (33) is satisfied, the fault-tolerant controller could be obtained, else if κ is less than a specified iterative times, set $\kappa = \kappa + 1$, go to Step 2, otherwise, it means no feasible solution could be found, EXIT.

$$|\text{Trace}(P_\kappa U + R_\kappa V + U_\kappa P + V_\kappa R) - n - n_r - n_v| \leq \varrho \tag{33}$$

4 A numerical example

In this section, the proposed reliable tracking control method is applied to a satellite system whose output follows a specific trajectory over network environment. Consider the following continuous-time plant [6]

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -0.09 & 0.09 & -0.04 & 0.04 \\ 0.09 & -0.09 & 0.04 & 0.04 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \omega(t) \tag{34}$$

$$y(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} x(t) \tag{35}$$

and the continuous-time reference model is

$$\dot{x}_r(t) = -x_r(t) + r(t) \tag{36}$$

$$y_r(t) = 0.5 x_r(t) \tag{37}$$

As shown in Fig. 1, the signals are transmitted via communication network with a sampling period 0.5 s. By discretising the above continuous-time systems, we can obtain the parameters in (1) as follows

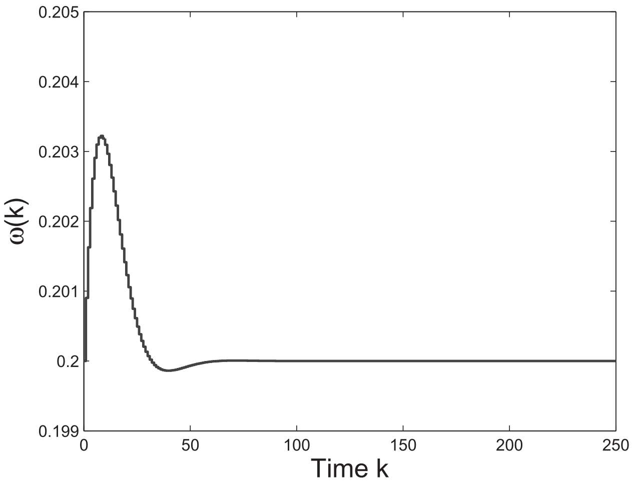

Fig. 3 *Trajectory of $\omega(k)$*

$$A = \begin{bmatrix} 0.9889 & 0.0111 & 0.4932 & 0.0068 \\ 0.0111 & 0.9889 & 0.0068 & 0.4932 \\ -0.0438 & 0.0438 & 0.9695 & 0.0305 \\ 0.0438 & -0.0438 & 0.0305 & 0.9695 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.1239 \\ 0.0011 \\ 0.4932 \\ 0.0068 \end{bmatrix}, \quad B_\omega = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

and the discretised reference model is

$$x_r(k+1) = 0.6065 x_r(k) + 0.3935 r(k) \tag{38}$$

$$y_r(k) = 0.5 x_r(k) \tag{39}$$

We assume $0 \leq d_k \leq 2$, and the disturbance $\omega(k) = 0.2 + 0.01e^{-0.1k}\sin(0.1k)$ whose trajectory is shown in Fig. 3; the input of reference model $r(k) = 4$; the initial condition of the satellite system is assumed to be $[-0.5 \ -0.3 \ 0.3 \ -0.3]^T$; and the initial condition of the reference model is 0.5.

In order to demonstrate the effectiveness of our proposed method, the following two cases listed in Table 1 are considered, and the corresponding controllers obtained from Theorem 1 together with the Algorithm by utilising the Matlab LMI Toolbox come next.

In Table 1, the controller of CASE 1 is called standard controller because of the system operating in a normal condition, while the controller of CASE 2 with consideration of SAFs is called fault-tolerant controller.

From Fig. 4, one can see obviously that the output of the plant (y) can track the output of the reference model (y_r)

Table 1 Controller parameters in different cases

CASE	Condition		Controller
1	$\bar{\Theta} = 1, \sigma = 0$	$\mathbb{K}_{SC}$	$K_1 = [-0.2154 \quad -0.0413 \quad -0.6324 \quad -0.4290]$ $K_2 = 0.0478$ $K_3 = -0.0041$
2	$\bar{\Theta} = 0.4, \sigma = 0.05$	$\mathbb{K}_{FTC}$	$K_1 = [-0.5308 \quad -0.1015 \quad -1.5685 \quad -1.0589]$ $K_2 = 0.1175$ $K_3 = -0.0101$

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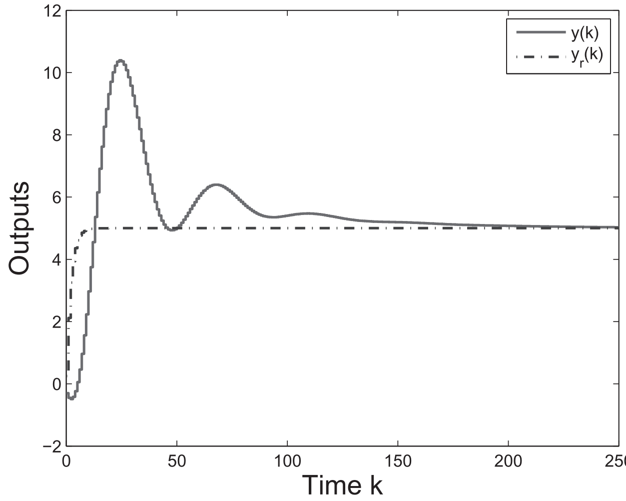

Fig. 4 *Outputs by using $\mathbb{K}_{SC}$ under Condition 1*

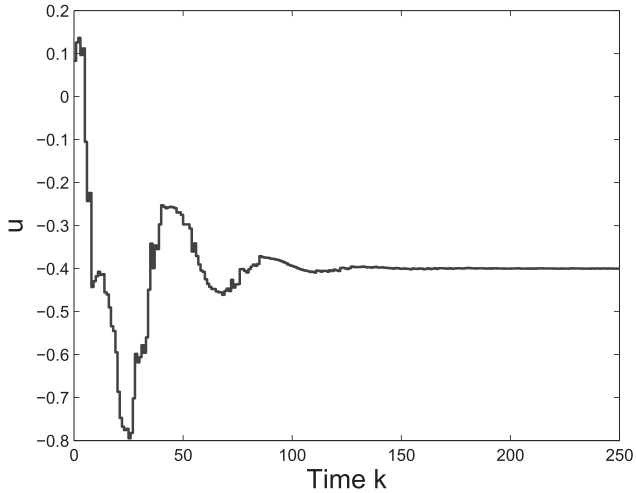

Fig. 5 *Control input $u(k)$ by using $\mathbb{K}_{SC}$ under Condition 1*

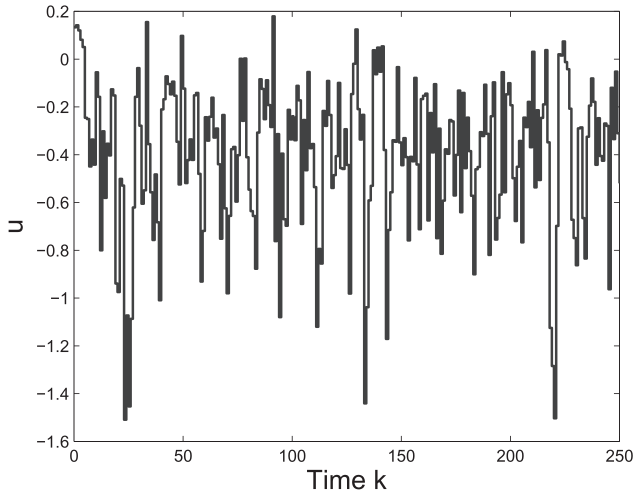

Fig. 6 *Control input $u(k)$ by using $\mathbb{K}_{FTC}$ under Condition 2*

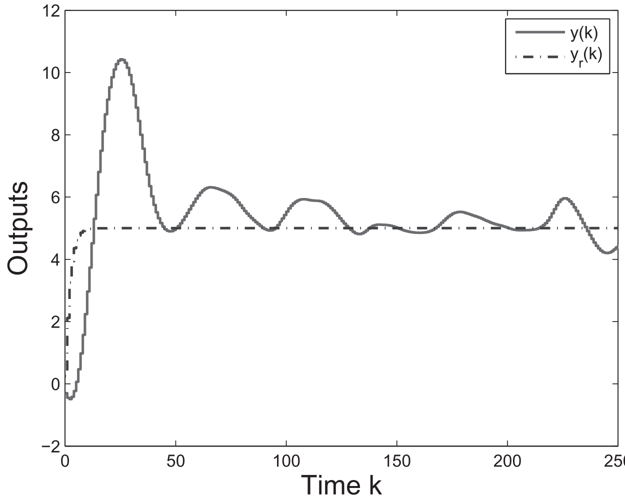

Fig. 7 *Outputs by using $\mathbb{K}_{FTC}$ under Condition 2*

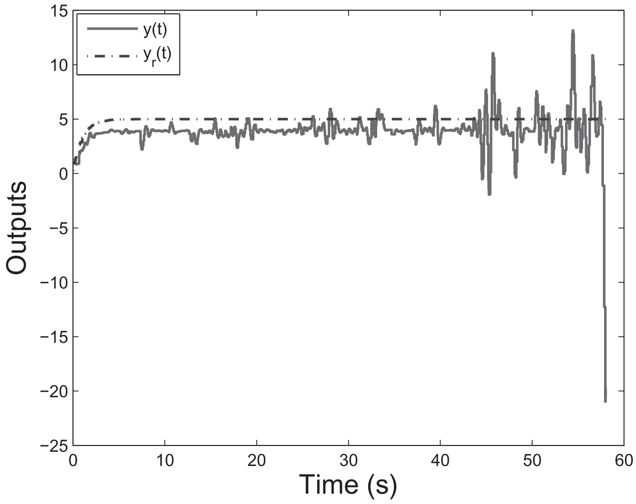

Fig. 8 *Outputs by using the controller in [6] under Condition 2*

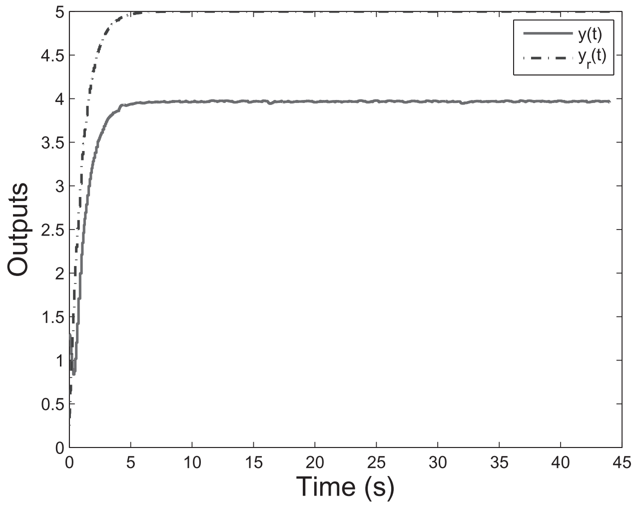

Fig. 9 *Outputs by using the controller in [6] under Condition 1*

precisely with zero SSE by using the SC in Case 1 listed in Table 1. Comparing Fig. 5 with Fig. 6, we can find that the actuator is affected by the SAFs, nevertheless, the output y can still track the output y_r precisely over the time under Condition 2, which is shown in Fig. 7. On the contrary, if the corresponding continuous-time system (34)–(36) by using the controller in [6] under Condition 2, it renders the system unstable (see Fig. 8). From this point of view, we conclude that it is necessary to design a fault-tolerant controller for the system, which is subject to SAFs.

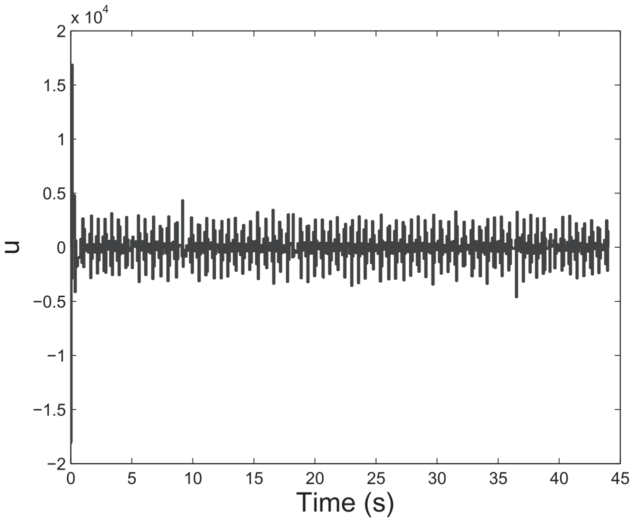

Fig. 10 *Control input $u(t)$ by using the controller in [6] under Condition 2*

Now we study the influence of the non-zero inputs. From Fig. 4, it can be seen that the steady tracking-output is of zero SSE by using our proposed method when the external inputs (ω and r) tend to non-zero constants, while a big SSE between the output y and y_r may be generated by using the method in [6] for system (34)–(36) under the same conditions (see Fig. 9). Since the criterion given in Theorem 1 is MSAS, the trajectory in Fig. 7 is oscillated near the reference output y_r under Case 2.

It should be pointed out that the big controller gains are not suggested in practice. since the actuator force will be in a saturated state while its amplitude is up to a certain lever. Moreover, it can arouse a tremendous noise as well, which could destabilise the system. Comparing the control force in Fig. 10 with the one in Figs. 5 and 6, one can see that the controller designed by our method is more suitable for design requirements.

5 Conclusion

This paper has investigated the problem of network-based fault-tolerant tracking control for the discrete-time system with both NEIs and SAFs. A new SAFs model is developed by introducing a rand diagonal matrix, which can characterise the actuator fault in each channels and cover several classes of well-studied fault model. By borrowing the idea of integrated control method, a novel control scheme is proposed, which can make the system be with zero SSE under NEIs. Based on Lyapunov function approach and cone complementary linearisation (CCL) algorithm, the controller parameters can be solved by LMI Toolbox easily. A numerical example is given to show the effectiveness of our proposed method.

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